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Author(s): Miao Yu & Deqing Huang

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Discrete-Time Adaptive Learning Control for Parametric Uncertainties with Unknown Periods

Miao Yu and Deqing Huang

Abstract—In this paper, we approach the problem of unknown periods for a class of discrete-time parametric nonlinear systems with nonlinearities which do not necessarily satisfy the sector-bounded condition. The unknown periods hide in the parametric uncertainties, which is difficult to estimate. By incorporating a logic-based switching mechanism, we estimate the period and bound of unknown parameter simultaneously under Lyapunov-based analysis. Rigorous proof is given to demonstrate that a finite number of switchings can guarantee the asymptotic regulation of the nonlinear system considered. The simulation result also shows the efficacy of the proposed switching periodic adaptive control method.

I. INTRODUCTION

Learning control is a powerful tool to deal with the tasks of tracking a given periodic desired trajectory or rejecting a periodic disturbance [1], [2], [3], [4], [5]. In the design of learning control, the signal period must be known *a priori*. From the perspective of internal model [6], without the exact knowledge of the period, the internal model cannot be constructed appropriately. Thus, most of the learning control methods based on internal model can not work.

The difficult problem associated with the unknown periods has attracted lots of attention during the past decades. Since the sinusoidal signal exists most commonly in the engineering practice and the signal processing, a number of references have been dedicated to estimating the frequency of a given sinusoidal signal. In [7], an adaptive notch filter is employed to form a period orbit finally, thus the frequency can be observed. Based on the adaptive notch filter, various methods are proposed to estimate the frequency of pure sinusoidal signal, sinusoidal signal with unknown phase, unknown magnitude and unknown bias [8], [9]. In [10], the authors proposed a global convergent frequency estimator and analyzed the influence on the converge speed.

By plugging the signal generator of the given signal into the system model, the problem of disturbance rejection of unknown period is dealt with for the nonlinear systems in [11], [12] under the framework of output regulation. The same idea was applied in the periodic disturbance rejection of optical disk drive [13].

In [14], the authors proposed a period identification algorithm based on the gradient minimization of an energy function for both continuous and discrete systems. By using

multiple memory-loops, [15] implemented a digital repetitive control design to attenuate the influence of variation of period-time. In [16], the problem of unknown periods was tackled by a switching estimation algorithm.

The concept of PAC arises in [17], [18], [19] due to the observation that the unknown periodic parameter remains a "constant" after a given period. Thus, the PAC approach updates the parameter estimate after the period interval, instead of carrying out the updating law continuously. Discrete PAC for parametric systems with sector-bounded nonlinearities has been presented in [18]. Although the PAC approach greatly improves the tracking performance compared with classical adaptive control, the linear growth condition on the nonlinearities largely restricts the classes of nonlinear systems that PAC can deal with. In the presence of non-sector nonlinearities, the main analysis tool for convergence proof in [18], the Key Technical Lemma ([20]), is no longer applicable. The continuous-time counterpart of this problem is that small-gain-type arguments also depend critically on the existence of sector-type bounds for the nonlinearities. In the continuous-time case, Lyapunov-based design and analysis was used to overcome this problem. Therefore, it is necessary to carry out PAC designs under a new framework. It has been stated in [21] that with an appropriate choice of nonlinear weighting coefficient in the weighted least squares form, a logarithmic-type Lyapunov function can be utilized to deal with the non-sector nonlinearities multiplying unknown constant parameters. Inspired by [21], a logarithmic-type discrete-time Lyapunov function is exploited to perform the stability analysis in this work. The nonlinear weighting coefficients and logarithmic form of Lyapunov function are designed accordingly to cope with the periodicity of unknown parameters. The Lyapunov-based analysis offers a deep insight for complex nonlinear systems with not only sector-bounded but also non-sector nonlinearities. Fig. 1([22]) shows the evolution of discrete adaptive control, which highlights the contributions of this work.

In this paper, we address the regulation problem for a class of discrete-time nonlinear systems with periodic parametric uncertainties whose period and bound are not known. A logic-based switching scheme is presented to estimate the unknown period and bound on-line. A continuous-time counterpart of switching control for unknown periods has been proposed in [16], where a periodic disturbance is considered. As is well known, the extension of algorithms from continuous-time systems to discrete-time systems is a non-trivial work, as the analysis tools are totally diffident. In addition, it is obvious that the problem of periodic distur-

Miao Yu is with Aalto University, Department of Biotechnology and Chemical Technology, P.O.Box 16100 FI-00076, Aalto, Finland. zjuyumiao@gmail.com

Deqing Huang is with Department of Aeronautics, Imperial College London, Prince Consort Road, London, SW7 2AZ, UK. elehd2012@gmail.com

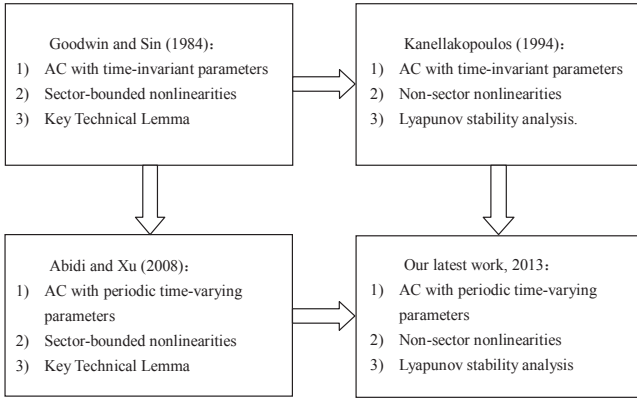


Fig. 1. ([22]) Evolution of adaptive control (AC) for discrete-time systems.

bance rejection could be treated as a special case considered in this paper. It is shown that the asymptotical regulation is ensured under a composition of Lyapunov-based adaptive control and the proposed switching logic.

The main contributions of this paper lie in:

- When the period and upper bound of unknown periodic parameter are known, we combine the results of [18] and [21] to develop a discrete-time PAC law for nonlinear systems with non-sector nonlinearities under a Lyapunov-based analysis.
- When the period and upper bound of unknown periodic parameter are unknown, we extend the switching learning control for continuous-time systems in [16] to discrete-time systems based on the proposed discrete-time PAC.
- Instead of estimating the period of an accessible signal, we consider a class of discrete-time nonlinear systems with non-sector nonlinearities and parametric parameters of unknown periods. The parameter is not accessible, moreover, none of the measurable signals such as the regulation error can reveal the periodic property of the parameters. Thus the period estimation methods on the single signal as [14], [10] are not be appropriate.
- The unknown parameter considered here is of a general type. It can be a sinusoidal signal, triangular wave, square wave, or any other shape with periodic property. Thus the internal model based algorithms which need the signal generator as [11], [12], [13] are no longer suitable.
- Discrete Lyapunov function is exploited to handle the nonlinear systems which contains non-sector nonlinearities. Thus the estimation methods in frequency domain [15] are not applicable.

The remainder of this paper is organized as follows: The PAC method for the nonlinear systems with non-sector nonlinearities is proposed in Section 2. Then the logic-based switching control design and rigorous proof concerning about the closed-loop stability are detailed in Section 3 to deal with the unknown period. In Section 4, an illustrating example is given. Finally Section 5 concludes the paper.

II. PERIODIC ADAPTIVE CONTROL FOR DISCRETE-TIME NONLINEAR SYSTEMS WITH NON-SECTOR NONLINEARITIES

In order to demonstrate the idea of this paper clearly, we consider the scalar discrete-time nonlinear system with only one unknown parameter as follows

$$x_{k+1} = \theta_k \xi_k + u_k \quad (1)$$

where k is the time index; x_k is the system state; u_k is the control input; θ_k is the unknown periodic parameter; $\xi_k = \xi(x_k)$ is the known nonlinear function which could be non-sector bounded; $x_0 = x(0)$ is the initial condition.

With respect to the unknown parameter θ_k , we have the following assumptions:

Assumption 1: θ_k is periodic with an unknown period $N > 1$, i.e. $\theta_{k+N} = \theta_k$.

Assumption 2: The upper bound ρ of θ_k , i.e. $\rho = \sup_{t \in [k, k+N-1]} |\theta_t|$, is unknown.

We would like to divide the control design into two steps. First, assume that the period and bound of unknown parameter θ_k are all known, we will first extend the periodic adaptive control (PAC) method in [18] to the nonlinear system (1) in the presence of non-sector nonlinearities. By incorporating the Lyapunov function into the PAC, the asymptotic regulation of system (1) could be ensured through rigorous proof. Then, in the next section we will introduce the logic-based switching mechanism to estimate the unknown factors online.

Under the framework of certainty equivalence, the periodic adaptive control law are designed as

$$u_k = -\hat{\theta}_k \xi_k \quad (2)$$

where the parameter estimate is updated periodically with a least-square form and a projection mechanism

$$\hat{\theta}_k = \begin{cases} \text{Proj}[\hat{\theta}_{k-N} + \alpha_k P_k \xi_{k-N} x_{k-N+1}], & k \in [N, \infty) \\ \text{Proj}[\hat{\theta}_0], & k = [0, N) \end{cases} \quad (3)$$

$$\text{Proj}[a] = \begin{cases} a, & |a| \leq \rho \\ \rho * \text{sgn}(a), & |a| > \rho \end{cases} \quad (4)$$

$$P_k = \begin{cases} P_{k-N} - \frac{\alpha_k P_{k-N}^2 \xi_{k-N}^2}{1 + \alpha_k P_{k-N} \xi_{k-N}^2}, & k \in [N, \infty) \\ P_0 > 0, & k = [0, N) \end{cases} \quad (5)$$

where α_k is a positive nonlinear weighting coefficient which will be defined later. From (5), it can be derived that for all $k \geq N$

$$P_k^{-1} = P_{k-N}^{-1} + \alpha_k \xi_{k-N}^2 \quad (6)$$

which implies that $P_k^{-1} \geq P_{k-N}^{-1} \geq P_0^{-1} > 0$ and $P_0 \geq P_{k-N} \geq P_k > 0$.

Define the parameter estimation error as $\tilde{\theta}_k = \theta_k - \hat{\theta}_k$. Substituting the PAC law (2) into system dynamic (1), it is easy to see that

$$x_{k+1} = \tilde{\theta}_k \xi_k \quad (7)$$

For the nonlinear system (1) with non-sector uncertainties, Key Technical Lemma, which is a common choice in

the least-square parameter identification, is not applicable. Hence it is necessary to carry out the PAC design under a new framework. Discrete-time Lyapunov stability analysis, though much more difficult due to its structure property, offers a deeper insight for the complex nonlinear systems with not only sector-bounded but also non-sector uncertainties.

Inspired by the discrete Lyapunov functions designed in [23], [21], [24], our Lyapunov function is formulated as

$$\begin{aligned} V_k &= \ln(1+x_{k-N+1}^2) + c \sum_{j=k-N+1}^k P_j^{-1} \tilde{\theta}_j^2 + \sum_{j=k-N+1}^k P_j^2 \\ &\triangleq V_{k,x} + cV_{k,\theta} + V_{k,P}, \quad k \in [N, \infty). \end{aligned} \quad (8)$$

The difference of the three components of Lyapunov function above will be analyzed separately. Considering the first part which is a logarithmic function of the square of the system state, the difference of $V_{k,x}$ can be expressed as

$$\begin{aligned} \Delta V_{k,x} &= V_{k,x} - V_{k-1,x} \\ &= \ln(1+x_{k-N+1}^2) - \ln(1+x_{k-N}^2) \\ &= \ln\left(\frac{1+x_{k-N+1}^2}{1+x_{k-N}^2}\right) \\ &= \ln\left(1 + \frac{-x_{k-N}^2 + x_{k-N+1}^2}{1+x_{k-N}^2}\right) \\ &\leq \frac{-x_{k-N}^2 + x_{k-N+1}^2}{1+x_{k-N}^2} \end{aligned} \quad (9)$$

where the last inequality follows from that $g > 0 \Rightarrow \ln g \leq g - 1$.

The difference of the second part $V_{k,\theta}$ which is concerned with parameter estimation errors is derived as

$$\begin{aligned} \Delta V_{k,\theta} &= V_{k,\theta} - V_{k-1,\theta} = P_k^{-1} \tilde{\theta}_k^2 - P_{k-N}^{-1} \tilde{\theta}_{k-N}^2 \\ &= P_k^{-1} (\theta_k - \text{Proj}[\tilde{\theta}_{k-N} + \alpha_k P_k \xi_{k-N} x_{k-N+1}])^2 \\ &\quad - P_{k-N}^{-1} \tilde{\theta}_{k-N}^2 \end{aligned} \quad (10)$$

Since $\theta_k \in [-\rho, \rho]$, from the definition of the projection function (4), it can be obtained that $|\theta_k - \text{Proj}[a]| \leq |\theta_k - a|$. Thus (10) yields

$$\begin{aligned} \Delta V_{k,\theta} &\leq P_k^{-1} (\tilde{\theta}_{k-N} - \alpha_k P_k \xi_{k-N} x_{k-N+1})^2 \\ &\quad - P_{k-N}^{-1} \tilde{\theta}_{k-N}^2 \\ &= (P_k^{-1} - P_{k-N}^{-1}) \tilde{\theta}_{k-N}^2 - 2\alpha_k \tilde{\theta}_{k-N} \xi_{k-N} x_{k-N+1} \\ &\quad + \alpha_k^2 P_k \xi_{k-N}^2 x_{k-N+1}^2 \\ &= -\alpha_k x_{k-N+1}^2 (1 - \alpha_k P_k \xi_{k-N}^2) \\ &= -\frac{\alpha_k x_{k-N+1}^2}{1 + \alpha_k P_{k-N} \xi_{k-N}^2} \leq 0 \end{aligned} \quad (11)$$

Regarding the P -matrix, since $P_0 \geq P_{k-N} \geq P_k > 0$, the difference of the third part $V_{k,P}$ is

$$\Delta V_{k,P} = V_{k,P} - V_{k-1,P} = P_k^2 - P_{k-N}^2 \leq 0 \quad (12)$$

Substituting (9)(11)(12) into the expression of ΔV_k , we obtain that

$$\begin{aligned} \Delta V_k &= \Delta V_{k,x} + c\Delta V_{k,\theta} + \Delta V_{k,P} \\ &\leq \frac{-x_{k-N}^2 + x_{k-N+1}^2}{1+x_{k-N}^2} - \frac{c\alpha_k x_{k-N+1}^2}{1+\alpha_k P_{k-N} \xi_{k-N}^2} \\ &= -x_{k-N+1}^2 \left(\frac{c\alpha_k}{1+\alpha_k P_{k-N} \xi_{k-N}^2} - \frac{1}{1+x_{k-N}^2} \right) \\ &\quad - \frac{x_{k-N}^2}{1+x_{k-N}^2} \end{aligned} \quad (13)$$

Then the kernel task is to find an appropriate positive nonlinear weighting coefficient α_k , such that

$$\frac{c\alpha_k}{1+\alpha_k P_{k-N} \xi_{k-N}^2} - \frac{1}{1+x_{k-N}^2} \geq 0 \quad (14)$$

Proposition 1: If the weighting coefficient is defined as

$$\alpha_k = 1 + \frac{\xi_{k-1}^2 \xi_k^2}{\xi_{k-N}^2}, \quad (15)$$

and the following inequality holds

$$\frac{P_0^{-1}}{\xi_k^2} + \xi_{k-1}^2 + \frac{\xi_{k-N}^2}{\xi_k^2} > \frac{1}{d} \quad (16)$$

where d is a positive constant which will be defined later, then with an appropriate choice of c , Inequality (14) holds for $k \geq N$.

Proof: From the updating law of P -matrix (5), we can see that

$$\begin{aligned} \frac{P_{k-N}^{-1}}{\xi_{k-N}^2} &= \frac{P_{k-2N}^{-1} + \alpha_{k-N} \xi_{k-2N}^2}{\xi_{k-N}^2} \\ &= \frac{P_{k-2N}^{-1}}{\xi_{k-N}^2} + \frac{\alpha_{k-N} \xi_{k-2N}^2}{\xi_{k-N}^2} \end{aligned} \quad (17)$$

Since $P_k^{-1} \geq P_{k-N}^{-1} \geq P_0^{-1} > 0$ and $\alpha_{k-N} = 1 + \frac{\xi_{k-N-1}^2 \xi_{k-N}^2}{\xi_{k-2N}^2}$, from (16), it follows that

$$\frac{P_{k-N}^{-1}}{\xi_{k-N}^2} \geq \frac{P_0^{-1}}{\xi_{k-N}^2} + \xi_{k-N-1}^2 + \frac{\xi_{k-2N}^2}{\xi_{k-N}^2} > \frac{1}{d} \quad (18)$$

Thus, from (18) it can be obtained that

$$\begin{aligned} \frac{1 + \alpha_k P_{k-N} \xi_{k-N}^2}{\alpha_k} &= \frac{1}{\alpha_k} + P_{k-N} \xi_{k-N}^2 \\ &\leq 1 + P_{k-N} \xi_{k-N}^2 < 1 + d \end{aligned} \quad (19)$$

since $\alpha_k \geq 1$. Choose $c > 1 + d$, then

$$\frac{c\alpha_k}{1 + \alpha_k P_{k-N} \xi_{k-N}^2} \geq \frac{c}{1 + d} > 1 \geq \frac{1}{1 + x_{k-N}^2} \quad (20)$$

Thus Inequality (14) holds. ■

Proposition 2: Inequality (16) holds with a suitable choice of d .

Proof: Choose an arbitrary positive constant δ_0 , then consider the following two cases:

1) $\xi_{k-1}^2 > \delta_0$. Inequality (16) holds with $d > \delta_0^{-1}$.

- 2) $\xi_{k-1}^2 \leq \delta_0$. From (5) and (11), we have $P_0^{-1} \tilde{\theta}_k^2 \leq P_k^{-1} \tilde{\theta}_k^2 \leq P_0^{-1} \tilde{\theta}_0^2$, thus $|\tilde{\theta}_k| \leq |\tilde{\theta}_0|$ for all $t \geq 0$. Then from (7) we have $|x_k| \leq |\tilde{\theta}_{k-1}| |\xi_{k-1}| \leq |\tilde{\theta}_0| \sqrt{\delta_0}$. Since x_k is bounded, the function ξ_k must be bounded by some constant as $\xi_k^2 \leq \delta_1$. Thus Inequality (16) holds with $d > \delta_1 P_0$.

Choose $d > \max\{\delta_0^{-1}, \delta_1 P_0\}$, then it is shown above that Inequality (16) always holds. ■

By the above two propositions, we have shown that for all initial conditions, there always exists a proper constant c as $c > 1 + \max\{\delta_0^{-1}, \delta_1 P_0\}$, such that the difference of Lyapunov function (13) remains nonpositive. Then the state x_k converges to zero as $k \rightarrow \infty$. We summarize our derivations in the following theorem:

Theorem 1: Consider the discrete-time nonlinear system (1), assuming that the period and upper bound of unknown parameter are all known, the adaptive control law (2), together with the periodic adaptation law (3), the projection function (4) and dynamic matrix updating law (5), ensures that the regulation error x_k converges to zero asymptotically.

Remark 1: It is not difficult to extend the design and analysis for scalar system (1) to the following system with multiple periodic unknown parameters

$$x_{k+1} = (\theta_k)^T \xi_k + u_k, \quad (21)$$

where $\theta = [\theta_1, \dots, \theta_m]^T \in \mathfrak{R}^m$ are the unknown periodic parameters; $\xi = [\xi_1, \dots, \xi_m]^T \in \mathfrak{R}^m$ are the known vector-valued nonlinear functions that do not necessarily satisfy the sector condition, ξ are bounded provided that x_k is bounded. The analysis is omitted due to space limitation.

III. SWITCHING LOGIC AND STABILITY ANALYSIS

The periodic adaptation law (3) and the projection function (4) make full use of the periodic and bounded property of the unknown parameter. However, the period and bound are not known in advance. Thus, seeking for the correct period and bound of θ_k remains a big challenge for the control design. In this section we shall utilize a logic-based switching mechanism to tune the unknown factors on-line. The switching process is described as follows:

Initialization:

- Preselect a positive constant σ .
- Set $l = 2$, $\mu = \sigma * l$, $t = 0$, $\hat{\theta}_j = 0, j = t, \dots, t + l - 1$, and $P_j = P_0, j = t, \dots, t + l - 1$.

Switching logic: For $k = [t, \dots, t + l - 1]$, $u_k = 0$. At each time $k \geq t + l$, if

$$x_{k-l+1}^2 > e^H - 1 \quad (22)$$

or if

$$\sum_{j=t+l}^k x_{j-l}^2 > H e^H \quad (23)$$

where $H = \ln(1 + x_0^2) + c l^3 \sigma^2 P_0^{-1} + l P_0^2$ is a constant, then we switch the period estimate l to $l + 1$. Reset the new bound estimate $\mu = \sigma * l$, initial conditions $t = k + 1$, $x_t = x_0$ and $P_j = P_0, j = t, \dots, t + l - 1$ with the new period estimate.

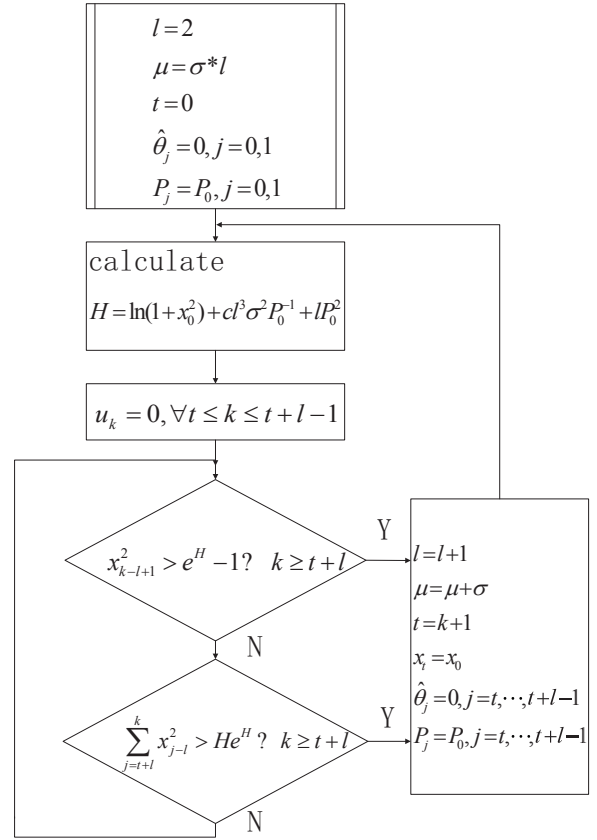


Fig. 2. Computer diagram of the switching algorithm

The learning process then repeats. The computer diagram is depicted in Fig. 2.

Theorem 2: Consider the discrete-time nonlinear system (1) under Assumptions (1) and (2). The adaptive control law (2) as well as the periodic adaptation law (3), the projection function (4) and dynamic matrix updating law (5), where the unknown period and upper bound of the unknown parameter are tuned by the switching logic proposed above, ensures that the regulation error x_k converges to zero asymptotically, i.e., $\lim_{k \rightarrow \infty} x_k = 0$.

Proof: We shall divide the whole proof into two parts. First, it is shown that if only a finite number of switching occur during the process, then the asymptotic regulation is achieved. Second, we shall show that indeed only a finite number of switchings can occur. Then the whole proof is completed.

Part 1: Suppose that at some time k_1 , the final i th switching occurs. Since after the i th switching, the estimation of unknown period is $i + 2$, according to the switching logic proposed above, we have

$$x_{k-i-1}^2 \leq e^H - 1, \forall k \geq k_1 + i + 2. \quad (24)$$

$$\sum_{j=k_1+i+2}^k x_{j-i-2}^2 \leq H e^H, \forall k \geq k_1 + i + 2. \quad (25)$$

where $H = \ln(1 + x_0^2) + c(i+2)^3 \sigma^2 P_0^{-1} + (i+2)P_0^2$. As $k \rightarrow \infty$, the sum of square of the system state remains bounded. Thus, it shows that the asymptotic regulation is guaranteed as k tends to infinity.

Part 2: Seeking a contradiction, suppose on the contrary that an infinite number of switchings would occur. Let the switching index i be an integer such that

$$i+2 \geq \frac{\rho}{\sigma} \quad (26)$$

$$\text{mod}(i+2, N) = 0 \quad (27)$$

which means that after the i th switching, the period estimate $i+2$ is a period of θ_k , the period estimate $(i+2) * \sigma$ is also an upper bound of θ_k . Therefore, Inequality $V_k - V_{k-1} \leq -x_{k-i-2}^2 / (1 + x_{k-i-2}^2)$ holds on the time interval $[k_1 + i + 2, k_2)$, where k_1 is the time when the i th switching happens, k_2 is the time when the next switching begins. Then for all $k \in [k_1 + i + 2, k_2)$, the following inequalities always hold

$$V_k \leq V_{k_1+i+1} \quad (28)$$

$$x_{k-i-2}^2 \leq (1 + x_{k-i-2}^2)(V_{k-1} - V_k) \quad (29)$$

By direct calculation

$$\begin{aligned} V_{k_1+i+1} &= \ln(1 + x_0^2) + c \sum_{j=k_1}^{k_1+i+1} P_j^{-1} \tilde{\theta}_j^2 + \sum_{j=k_1}^{k_1+i+1} P_j^2 \\ &\leq \ln(1 + x_0^2) + c(i+2)P_0^{-1} \rho^2 + (i+2)P_0^2 \\ &\leq \ln(1 + x_0^2) + c(i+2)^3 \sigma^2 P_0^{-1} + (i+2)P_0^2 \\ &\triangleq H \end{aligned} \quad (30)$$

As a consequence,

$$\begin{aligned} x_{k_2-i-2}^2 &\leq \exp(V_{k_2-1}) - 1 \\ &\leq \exp(V_{k_1+i+1}) - 1 \leq e^H - 1 \end{aligned} \quad (31)$$

$$\sum_{j=k_1+i+2}^{k_2-1} x_{j-i-2}^2 \leq \sum_{j=k_1+i+2}^{k_2-1} (1 + x_{j-i-2}^2)(V_{j-1} - V_j) \quad (32)$$

Since $(1 + x_{j-i-2}^2) \leq \exp(V_{j-1}) \leq \exp(V_{k_1+i+1})$, from the positiveness of V_{k_2-1} , we have

$$\begin{aligned} \sum_{j=k_1+i+2}^{k_2-1} x_{j-i-2}^2 &\leq (V_{k_1+i+1} - V_{k_2-1}) \exp(V_{k_1+i+1}) \\ &\leq H e^H \end{aligned} \quad (33)$$

It have been shown that none of the switching conditions (24) or (25) holds at the time instant $k_2 - 1$. Thus no switching occurs at the time instant k_2 . A contradiction exists, which means that only a finite number of switchings could occur.

That completes the whole proof. \blacksquare

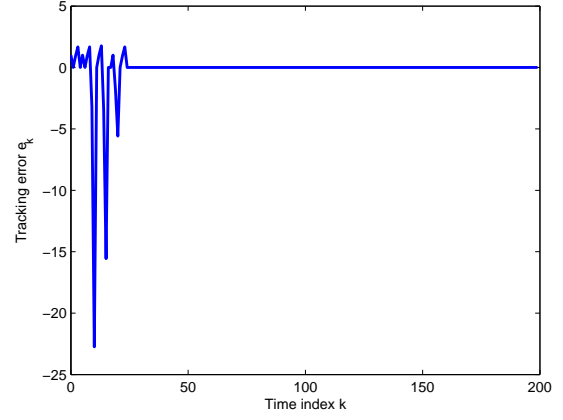


Fig. 3. Tracking error of discrete-time system.

IV. ILLUSTRATIVE EXAMPLE

Consider the following discrete-time nonlinear system

$$x_{k+1} = \sin(2\pi k/5)(x_k^2 + |x_k| + 1) + u_k, \quad (34)$$

where $\theta_k = \sin(2\pi k/5)$, the known nonlinear function is $x_k^2 + |x_k| + 1$, which does not satisfy the sector condition. The unknown period $N = 5$, the upper bound $\eta = 1$. The reference trajectory is $x_{d,k} = 10 \sin(4\pi k)$. In the simulation, the parameters are chosen as follows: $P_0 = 200$; $b = 30$; $\sigma = 0.2$; $x_0 = 1$. The simulation results are shown in Fig. 3- Fig. 5. The switching index increases until it arrives at an ideal value $\hat{N} = 5$.

The simulation result with increment $\sigma = 0.1$ is depicted in Fig. 6, where other parameters remain unchanged. In this case, when the switching index arrives at a minimum ideal value $\hat{N} = 5$, the upper bound estimate $\hat{\eta} = \hat{N} * \sigma = 0.5$, which is below the actual value $\eta = 1$. Thus the PAC law cannot work well. The switching index continues increasing until it arrives at the next ideal value $\hat{N} = 10$. Then the upper bound estimate $\hat{\eta} = \hat{N} * \sigma = 1$, which is not below the actual bound. In consequence, the PAC law works well and the switching index remains at $\hat{N} = 10$.

In the derivation of the stability analysis of the proposed switching learning control, we can see that the threshold value H is relatively conservative. Since we simply set $u = 0$ during the first period, a large transient performance may appear, as seen in Fig. 3. In order to overcome this difficulty, we can adopt a traditional feedback control in addition to PAC law, which suppresses the undesirable large peak.

V. CONCLUSION

A switching periodic adaptive control scheme has proposed for discrete-time parametric systems with nonlinearities of unknown period and bound. With a suitable choice of a nonlinear weighting coefficient, a weighted Least-Squares form is utilized to periodically update the parameter estimates. Unknown period and bound are estimated by a logic-based switching mechanism online.

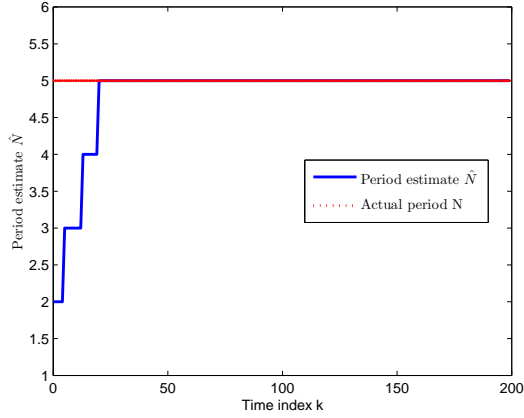


Fig. 4. Period estimate of discrete system with $\sigma = 0.2$.

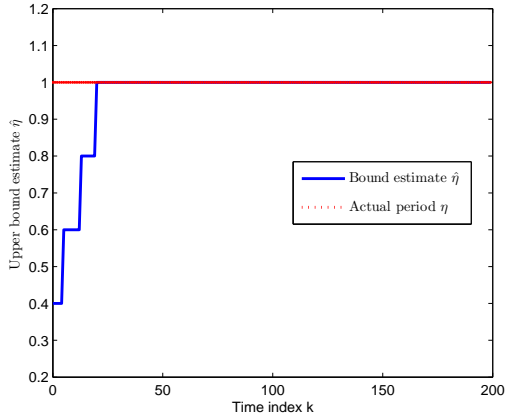


Fig. 5. Bound estimate of discrete system with $\sigma = 0.2$.

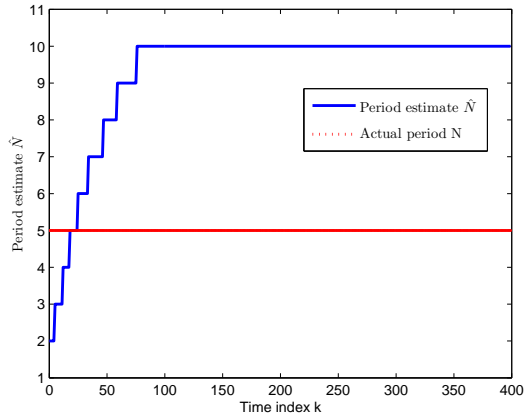


Fig. 6. Period estimate of discrete system with $\sigma = 0.1$.

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